

11.1 SEQUENCES

$$\begin{aligned}\text{eg. } \{2n+3\}_{n=4}^{\infty} &= \{2 \cdot 4+3, 2 \cdot 5+3, 2 \cdot 6+3, 2 \cdot 7+3, \dots\} \\ &= \{11, 13, 15, 17, \dots\}\end{aligned}$$

$$a_n = 2n+3, \quad n \geq 4$$

$$a_n = f(n), \quad n \geq n_0$$



**GENERAL
FORMULA**

**RECURRENCE
RELATION**



$$a_n = a_{n-1} + 2, \quad n \geq 2$$

$$a_1 = 11$$

$$n=2: a_2 = a_1 + 2 = 11 + 2 = 13$$

$$n=3: a_3 = a_2 + 2 = 13 + 2 = 15$$

$$n=4: a_4 = a_3 + 2 = 15 + 2 = 17$$



A SEQUENCE
IS A FUNCTION

WITH DOMAIN RESTRICTED

TO INTEGERS GREATER OR

EQUAL TO A CERTAIN INTEGER

$$\text{AND } a_1 = 11$$



**INITIAL
VALUE/CONDITION**

$$\{11, 13, 15, 17, \dots\}$$

**RECURSIVE
DEFINITION
OF A SEQUENCE**

$$a_{n+1} = a_n + 2, n \geq 1 \text{ AND } a_1 = 11$$

$$a_1 = 11$$

$$n=1: a_2 = a_1 + 2 = 11 + 2 = 13 \dots$$

$$\{a_n\}$$

DEFINITION: A SEQUENCE $\{a_n\}$ IS BOUNDED ABOVE ^{BELOW} IF THERE IS A NUMBER M SUCH THAT $a_n < M$ FOR ALL $n \geq 1$

$$a_n > m$$

A SEQUENCE IS BOUNDED IF IT IS BOUNDED ABOVE AND BELOW

$$\text{eg. } \{2-3n\} = \{-1, -4, -7, -10, \dots\}$$

↑ NOTE: LINEAR FUNCTION

DECREASING

SO ALL TERMS ^{LESS THAN} ~~SMALLER~~ THE FIRST TERM

SEQUENCE IS BOUNDED ABOVE

$$a_n \leq -1$$

$$a_n < 0$$

↑ M

★ $-\infty$ AND ∞ ARE NOT NUMBERS

$\{2-3\cos n\}$ IS BOUNDED

(IMPLIED "BOUNDED ABOVE
+ BELOW")

$$-1 \leq \cos n \leq 1$$

$$3 \geq -3\cos n \geq -3$$

$$5 \geq 2-3\cos n \geq -1$$

$$-1 \leq 2-3\cos n \leq 5$$

$$-2 < a_n < 6 \quad -1.1 < a_n < 5.1$$

$\uparrow$
 m

$\uparrow$
 M

DEFINITION: A SEQUENCE $\{a_n\}$ IS **DECREASING** IF $a_n > a_{n+1}$ FOR ALL $n \geq 1$
OR **INCREASING** IF $a_n < a_{n+1}$ FOR ALL $n \geq 1$

A SEQUENCE IS MONOTONIC IF IT IS EITHER INCREASING
OR DECREASING

$$\left\{ \frac{2n+1}{5n-4} \right\}$$

$$a_n - a_{n+1} = \frac{2n+1}{5n-4} - \frac{2(n+1)+1}{5(n+1)-4}$$

$$= \frac{2n+1}{5n-4} - \frac{2n+3}{5n+1}$$

$$= \frac{(2n+1)(5n+1) - (2n+3)(5n-4)}{(5n-4)(5n+1)}$$

$$= \frac{(\cancel{10n^2} + 7n + 1) - (\cancel{10n^2} + 7n - 12)}{(5n-4)(5n+1)}$$

$$= \frac{13}{(5n-4)(5n+1)} > 0$$

$$n \geq 1$$

$$a_n - a_{n+1} > 0, \quad n \geq 1$$

$$\text{I.E. } a_n > a_{n+1}, \quad n \geq 1$$

SEQUENCE IS DECREASING

(IS MONOTONIC)

$$f(x) = \frac{2x+1}{5x-4}$$

IS CONTINUOUS FOR $x \geq 1$

IE. ON $[1, \infty)$

$$f'(x) = \frac{2(5x-4) - (2x+1)(5)}{(5x-4)^2}$$

$$= \frac{10x-8-10x-5}{(5x-4)^2}$$

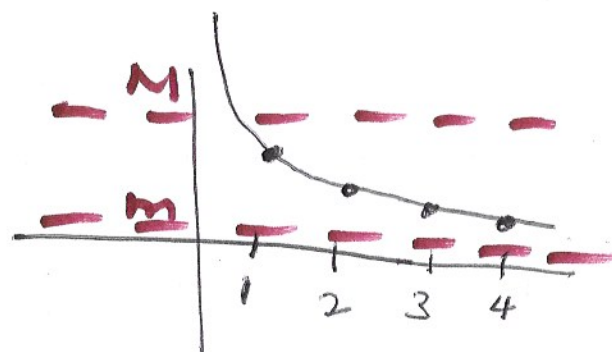
$$= \frac{-13}{(5x-4)^2} < 0$$

IE. $f(x)$ IS DECREASING FOR $x \geq 1$

$f(n)$ " " " $n \geq 1$

IE. ~~$f(x)$~~

$\left\{ \frac{2n+1}{5n-4} \right\}$ IS DECREASING



NOTE:

SEQUENCE IS
BOUNDED